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LINEARIZED EQUATIONS OF PROJECTILE MOTION INCLUDING WIND EFFECTS

by

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ABSTRACT. The linearized equations of motion of a yawing projectile are generalized to include the effects of wind, and expressed in a form suitable for use on an analog computer. They should be especially useful in interpreting the data obtained with yaw sondes.

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FOREWORD

This work is part of a continuing program to provide more complete and more versatile tools for studying projectiles and rockets and assisting in design of weapons.

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INTRODUCTION

Reference 1 presents an analysis of the yaw of certain artillery shell. The angular motion had been measured by solar yaw sondes, and was compared with the motion predicted by theory. During the comparison, the parameters in the theory, especially the aerodynamic coefficients, were varied until good fits were obtained between data and calculations. Similar studies of what is essentially the 7-caliber Army-Navy shaped projectile are reported more briefly in Ref. 2 and 3. Now digital computer programs, in several versions, are available which solve the dynamical equations of motion, and these equations are complete and rigorous insofar as the aerodynamic force system is known. But up to the present the turn-around time on available digital machines has been a few hours at the least. Furthermore, automatic iterative methods of parameter selection appear to be excluded, for they require a starting solution which is already a very good fit to the data. If, for example, there is anywhere a discrepancy in the phase of oscillation of even half a cycle, an iteration will almost certainly give no or a nonsensical result. The intervention of human judgment, guided by considerations such as those stemming from Dr. C. H. Murphy's "quasi-linear" analysis, has, so far, been essential. Hence an analog computer has been used, which computes the motion in about one-and-a-half times the real time of flight and provides concurrently a plot which can be directly compared with the plotted data. The limitations of existing analog devices have required, in turn, a simplification of the equations of motion.

The simplification consists in breaking the problem into two parts. The first part is the computation of a smoothed trajectory, essentially that of a particle, neglecting oscillations. (Spin history is also fitted here.) In the second part the transverse angular motion is computed from equations which have been simplified to the extent that the spherical trigonometry of the rotation group representation is linearized. On the other hand, nonlinearities in the aerodynamic force and moment system are specifically included, so far as known or suspected. This part uses the velocity, trajectory angle, and altitude (via air density) from the first part. Feedback of the oscillations into the first part is <u>not</u> permitted. An apparent exception to the last statement arises because it has sometimes been desirable in the first part, to take account of the effect of yaw on drag. But this has been done by inserting a "mean square yaw" as a smooth, known function of the time, its values having been chosen by estimation with the aid of a preliminary fit of the oscillations.

The system of equations used so far makes no provision for the effects of wind. As mentioned in Ref. 1, there are clear indications that the system needs such. In addition to making the oscillation fit better and more realistically, inclusion of wind ought to give a more realistic and more easily fitted particle trajectory. These points were again raised recently during a visit to this Station by people from the Ballistics Research Laboratory, Aberdeen Proving Ground. Now it was not immediately obvious how to include the wind terms in the equations in a way most nearly consistent with the levels of approximation used before.

This note proposes, and gives a skeleton derivation of, a set of equations which include wind. Two cautions are in order: first, the equations have not yet been tested in actual application; second, the approximations would be inadequate when yaw becomes large or the wind speed an appreciable fraction of the projectile speed. Such conditions could arise, for example, near the summit of a very high-angle trajectory.

THE EQUATIONS

Two comments are in order. Because a so-called yaw sonde or yaw camera actually measures not yaw but the angle between the projectile axis and a line to the sun, we must keep track of the actual orientation as well as the yaw. This requires, for the angular motion, a system of equations of higher order than that usually used in analytical discussions of stability. Then we wish the particle or reference trajectory to be as close to the real one as possible. This will require including wind in both of the normal equations. This, in turn, together with our desire to be able to include mean square yaw effects on drag means that the natural aerodynamic coefficients to use are K_D and K_L, not K_{DA} and K_N, and furthermore, that the particle trajectory is to be calculated neglecting the odd powers of the yaw <u>as measured in the air mass</u>.

We will use reduced velocity $U = V/d$ and wind $W = (\text{Wind velocity})/d$ where d is reference diameter.

Let

$$U_r = U - W, \quad u = |U|, \quad u_r = |U_r|.$$

$$W = i\, w_1 + j\, w_2$$

We define the aerodynamic coefficients and, implicitly, the coordinate system by taking P as a unit vector along the projectile axis, and writing

$$P = i \cos\gamma \cos\phi_2 + j \sin\gamma \cos\phi_2 + k \sin\phi_2$$

$$\gamma = \alpha + \phi_1$$

$$U = u(i \cos\beta \cos\theta_2 + j \sin\beta \cos\theta_2 + k \sin\theta_2)$$

$$\beta = \alpha + \theta_1$$

$$\dot{U} = f/dm - jg/d$$

$$f/d = -\rho d^3 \left[K_D u_r U_r + K_L U_r x(U_r xP) + K_F \omega(U_r xP)\right]$$

or

$$\dot{U} = -Du_r U_r - LU_r x(U_r xP) - F\omega(U_r xP) - jg/d.$$

The spin angular velocity is ω, and

$$D = \rho d^3 K_D/m,\ L = \rho d^3 K_L/m,\ F = \rho d^3 K_F/m.$$

Now $u\dot{u} = U \cdot \dot{U}$.

$$U \cdot U_r = u\, u_r \cos\Gamma = u\, u_r \left[1 - O(|W|^2)\right] \simeq u\, u_r$$

Also we set $P = U_r \cos\delta_r/u_r + \Delta_r$ with $U_r \cdot \Delta_r = 0$ and $|\Delta_r| = \sin\delta_r$.

Then $U_r xP = U_r x\Delta_r$ and $U \cdot (U_r xP) = U \cdot (-Wx\Delta_r)$.

The term in F of $u\dot{u}$ will then both be small and of first order in yaw, and is to be neglected.

$$U_r x(U_r xP) = U_r x(U_r x\Delta_r) = -U_r^2 \Delta_r$$

The term in L is, then, also first order in yaw and should be neglected. Therefore, we take (neglecting also θ_1)

$$u\dot{u} \simeq -Du_r^2 u - g \sin\alpha/d \qquad (1)$$

Now

$$\dot{U} = \dot{u}\,(i \cos\beta \cos\theta_2 + j \sin\beta \cos\theta_2 + k \sin\theta_2)$$
$$+ u\dot{\beta} \cos\theta_2 (-i \sin\beta + j \cos\beta)$$
$$+ u\dot{\theta}_2 (-i \cos\beta \sin\theta_2 - j \sin\beta \sin\theta_2 + k \cos\theta_2)$$

$$u\dot{\beta} \cos\theta_2 = \dot{U} \cdot (-i \sin\beta + j \cos\beta) = \dot{U} \cdot \epsilon_1$$

$$u\dot{\theta} = \dot{U} \cdot (-i \cos\beta \sin\theta_2 - j \sin\beta \sin\theta_2 + k \cos\theta_2)$$
$$= \dot{U} \cdot \epsilon_2$$

Now

$$U \cdot \epsilon_1 = U \cdot \epsilon_2 = 0$$

$$\therefore\ U_r \cdot \epsilon_1 = -W \cdot \epsilon_1 = w_1 \sin\beta \simeq w_1 \sin\alpha$$

$$U_r \cdot \epsilon_2 = -W \cdot \epsilon_2 = w_1 \cos\beta \sin\theta_2 - w_2 \cos\theta_2 \simeq -w_2$$

$$U_r x(U_r xP) = -u_r^2 P + u_r U_r \cos\delta_r$$

$$[U_r x(U_r xP)] \cdot \epsilon_1 = -u_r^2 P \cdot \epsilon_1 + u_r \cos\delta_r\, U_r \cdot \epsilon_1$$

$$[U_r x(U_r xP)] \cdot \epsilon_2 = -u_r^2 P \cdot \epsilon_2 + u_r \cos\delta_r U_r \cdot \epsilon_2.$$

But

$$P \cdot \epsilon_1 = \cos\phi_2 \sin(\gamma - \beta) \simeq \phi_1 - \theta_1$$

$$P \cdot \epsilon_2 = -\cos(\gamma - \beta)\sin\theta_2 \cos\phi_2 + \sin\phi_2 \cos\theta_2 \simeq \phi_2 - \theta_2$$

Or

$$[U_r x(U_r xP)] \cdot \epsilon_1 \simeq u_r^2(\phi_1 - \theta_1) + u_r w_1 \sin\alpha$$

$$[U_r x(U_r xP)] \cdot \epsilon_2 \simeq -u_r^2(\phi_2 - \theta_2) - u_r w_2 .$$

By straightforward expansion followed by linearization we find

$$(U_r xP) \cdot \epsilon_1 \simeq -u(\phi_2 - \theta_2) - w_2$$

$$(U_r xP) \cdot \epsilon_2 \simeq u(\phi_1 - \theta_1) - w_1 \sin\alpha$$

Thus

$$u\dot{\beta} \simeq -Du_r w_1 \sin\alpha + L u_r \left[u_r(\phi_1 - \theta_1) - w_1 \sin\alpha\right]$$
$$+ F\omega\left[u(\phi_2 - \theta_2) + w_2\right] - \frac{g\cos\beta}{d}$$

but

$$\dot{\beta} = \dot{\alpha} + \dot{\theta}_1 \quad , \quad \cos\beta \simeq \cos\alpha - \theta_1 \sin\alpha$$

We take

$$u\dot{\alpha} = -g\cos\alpha/d - Du_r w_1 \sin\alpha \tag{2}$$

Then

$$u\dot{\theta}_1 \simeq Lu_r^2\,\delta_{1r} + F\omega u\delta_{2r} - \theta_1 g \sin\alpha/d \tag{3a}$$

with

$$\delta_{1r} = \phi_1 - \theta_1 - w_1 \sin \alpha/u \quad (\text{since } u/u_r \simeq 1)$$

$$\delta_{2r} = \phi_2 - \theta_2 + w_2/u$$

And

$$u\,\theta_2 = Du_r w_2 + Lu_r^2\,\delta_{2r} - F\omega u\,\delta_{1r} \tag{3b}$$

The Magnus force terms (with F as a factor) have been included only for completeness. It is recommended that they not be used.

The torque is taken to be

$$\tau = -\rho d^5 K_A u_r \omega P + \rho d^5 K_M u_r (U_r x P)$$

$$- \rho d^5 K_T \omega P x (U_r x P) - \rho d^5 K_H u_r \Omega$$

where Ω is the transverse angular velocity and $\Omega \cdot P = 0$.

If I_p and I_T are the polar and transverse moments of inertia, we have

$$\tau = \frac{d}{dt}\left[I_p \omega P + I_T \Omega\right]$$

$$= I_p \dot{\omega} P + I_p \omega \dot{P} + I_T \dot{\Omega}$$

The easiest way to find Ω is to note that $\dot{P} = \Omega x P$, and hence $P x \dot{P} = P x (\Omega x P) = \Omega$.

We find, to the approximation we need

$$\Omega \simeq \dot{\phi}_2 (i \sin \gamma - j \cos \gamma) + k \dot{\gamma},$$

$$\dot{\Omega} = O(\dot{\phi}_2^2 + \dot{\gamma}^2),$$

and

$$\dot{P} \simeq -\dot{\gamma}(i \sin \gamma - j \cos \gamma) + k \dot{\phi}_2.$$

But

$$P \cdot \dot{P} = 0$$

or

$$I_p \dot{\omega} \simeq \tau \cdot P$$

$$\dot{\omega} \simeq -Au_r \omega \tag{4}$$

with $A = \rho\, d^5\, K_A/I_p$.

But

$$\dot{\Omega} + \bar{\omega}\,\dot{P} = [\tau - P(\tau \cdot P)]/I_T = \tau'/I_T$$

where

$$\bar{\omega} = \omega\, I_p/I_T$$

Neglecting $\ddot{\alpha}$ and terms in $\dot{\phi}_2\,\dot{\gamma}$, and setting $\epsilon_3 = i \sin\gamma - j \cos\gamma$,

$$\ddot{\phi}_2 - \bar{\omega}\,\dot{\phi}_1 \simeq \omega\,\dot{\alpha} + \tau' \cdot \epsilon_3/I_T$$

$$\ddot{\phi}_1 + \bar{\omega}\,\dot{\phi}_2 \simeq \tau' \cdot k/I_T.$$

Expanding the cross products as before,

$$\ddot{\phi}_1 + \bar{\omega}\,\dot{\phi}_2 \simeq - Hu_r\,\dot{\phi}_1 + Mu_r u\,\delta_{1r} + T\bar{\omega}u\,\delta_{2r} \tag{5a}$$

(here $H\dot{\alpha}$ is neglected)

$$\ddot{\phi}_2 - \bar{\omega}\,\dot{\phi}_1 \simeq - Hu_r\dot{\phi}_2 + Mu_r u\,\delta_{2r} - T\bar{\omega}u\,\delta_{1r}$$

$$- \frac{\omega}{u}\left(\frac{g \cos\alpha}{d} + Du_r\,\omega_1 \sin\alpha\right) \tag{5b}$$

One should be able to drop the distinction between u and u_r in all terms <u>except</u>:

1. The drag terms in Eq. 1
2. The gravity term of Eq. 2
3. Equation 4
4. The main moment term (that in M) in Eq. 5

The last item (4) is not really an inconsistency, for the difference between u and u_r will affect the frequency of the precessions.

The coordinates of the particle trajectory are, of course, obtained from

$$\left.\begin{aligned} \dot{x} &= d\,u\,\cos\alpha \\ \dot{y} &= d\,u\,\cos\alpha \end{aligned}\right\} \tag{6}$$

The numbered equations form the desired system. The dimensionless aerodynamic coefficients are proportional to the relative air density ρ/ρ_o.

M, T, H, L, and F, if used, may be taken as functions of Mach number = U_r/a, and of instantaneous $\delta_r^2 = \delta_{1r}^2 + \delta_{2r}^2$. Dependence on $\dot{\phi}_1$ and $\dot{\phi}_2$ may also be included if desired, so long as the well-known symmetry conditions are satisfied. D will, as mentioned above, depend on Mach number and a smoothed version of δ_r^2, chosen before the particular angular motion calculation. A probably needs to depend only on Mach number.

REFERENCES

1. Naval Weapons Center. Yawing Motion of 5"0 Mk 41 Projectile Studied by Means of Yaw Sondes, by W. R. Haseltine. China Lake, Calif., NWC, August 1969. 48 pp. (NWC TP 4779.)

2. ______. Yawing of BOMROC: (Fitting Predicted Motion to Data from Yaw Sonde), by W. R. Haseltine. China Lake, Calif., NWC, September 1965. 11 pp. (TN 507-14.)

3. ______. Yawing of BOMROC, by W. R. Haseltine. China Lake, Calif., NWC, December 1965. 8 pp. (TN 507-15.)

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